Control of Spin Ambiguity During Reorientation of an Energy Dissipating Body

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A quasi-rigid body initially spinning about its minor prinicpal axis and experiencing energy dissipation will enter a tumbling mode and eventually reorient itself such that stable spin about its major principal axis is achieved. However, in this final state the body may be spinning in a positive or negative sense with respect to its major axis and aligned in a positive or negative sense with the inertially fixed angular momentum vector. This ambiguity can be controlled only through an active system. The associated dynamical formulations and simulations of uncontrolled reorientations are presented. Three control schemes are discussed and results offered for specific examples. These schemes include displacement of internal masses, spinning up of internal inertia, and reaction jets, all of which have demonstrated the ability to control spin ambiguity.

Nomenclature

A, B, C = fluid slug moments of inertia about 1, 2, 3, axes, respectively
$\mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3$ = units vectors along principal body axes
D = diameter of ring cross section
f = Darcy-Wiesbach resistance coefficient
H = magnitude of angular momentum
I_1, I_2, I_3 = principal moments of inertia
m = mass of each moving mass
N_3 = moment due to viscous shear in ring
P = circumference of the ring cross section
r, R = parameters of moving mass system defined in Fig. 3
R_d = mean radius of viscous ring
R_n = Reynolds Number = Dv/V
T = rotational kinetic energy
v = velocity of the fluid slug relative to the ring
$\alpha, \dot{\alpha}$ = angular position and velocity of fluid slug relative to the ring, respectively
β = angular span of fluid slug in the ring
ν, ρ = kinematic viscosity and density of damper fluid, respectively
τ_0 = shear stress
θ = nutation angle measured between $\mathbf{d}_3$ and the angular momentum vector
$\boldsymbol{\omega}$ = angular velocity

Subscripts

a, b = after and before separatrix crossing, respectively
u, l = upper and lower limits, respectively

Introduction

A SPINNING quasi-rigid body with energy dissipation, free of applied torques and active controls, is stable only when rotating about its axis of maximum inertia. If this body is initially spinning about its minor principal axis it will enter a tumbling mode and eventually reorient itself such that stable spin about its major principal axis is achieved. In this final state the positive major axis can be aligned with the angular momentum vector in either of two ways, distinguished by a 180° rotation about a transverse axis. This ambiguity is important when yo-yo devices are installed to despin a vehicle after establishing a stable spin state, e.g., the ATS-V case.[1] Large-angle reorientation may involve pointing an apogee kick motor for orbit circularization.[2] This requires proper alignment of the major axis and angular momentum vector. The ambiguity responsible for this uncertainty is the result of uncontrolled transition from spin with nutation and precession about the minor axis to spin with simlar oscillations about the major axis. Prediction of transition dynamics for a given body depends on initial spin conditions and precise modelling of the energy dissipation characteristics, which is generally not possible. Therefore, final spin direction is unpredictable and can be controlled only though an active system. The associated dynamical formulations and simulations of uncontrolled reorientations are presented. Three control schemes are discussed and results offered for specific examples. These techniques include displacement of internal masses, spinning up of internal inertia, and reaction jets, all of which have demonstrated the ability to control spin ambiguity.

Effects of energy dissipation on satellite attitude motion are well-known and have been employed for passive stabilization[3] and reorientation[4] when the sense of spin with respect to body axes was not important and when the situation was not anticipated. Many investigations have dealt with symmetric and nonsymmetric satellite control using energy dissipation and addition,[5] but no prior attempt has considered control of the spin ambiguity.

Nature of Torque-Free Motion with Dissipation

Constant energy motion of a nonsymmetric, rigid body in a torque-free environment is geometrically described by Poinsot's solution.[6] Motion is represented as rolling without slip of an inertia ellipsoid (i.e., the ellipsoid semiaxes are inversely proportional to the square roots of the principal body inertias taken through the center of mass) on an inertially fixed plane with the body center of mass located at the ellipsoid center and remaining a fixed distance $(2T/H)^{1/2}$ from the plane. This ellipsoid is body-fixed and its contact point at the "invariable plane" represents the instantaneous axis of body rotation. The locus of all contact points on the ellipsoid is a closed path called a polhode. If the body is quasi-rigid and experiences energy dissipation, the polhode path is not closed and the distance to the centroid diminishes until a minimum energy state is reached, corresponding to steady spin about the major

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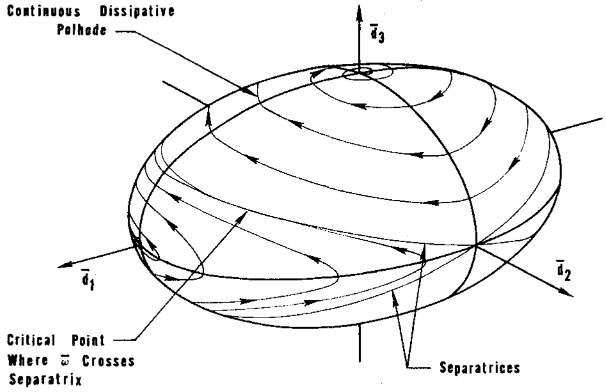

Fig. 1 Polhode of dissipative motion.[6]

axis. The polhode form for dissipative motion beginning near the minor axis of a nonsymmetrical body is illustrated in Fig. 1. The separatrices represent boundaries between major and minor axis rotational motion. As the polhode crosses one of these two boundaries motion is transformed from spin with wobble about the minor axis to spin with wobble about the positive or negative major axis, depending on which separatrix is crossed. Thus, the ambiguity of concern here is associated with the sign of $\boldsymbol{\omega} \cdot \mathbf{d}_3$ after a separatrix is crossed.

Analytical interpretation of polhode motion is handled through the use of nutation angle θ. The separatrices correspond to an energy level of $H^2/2I_2$. Solutions of motion are not of particular interest here, since only the relationship between energy level θ and the sign of $\boldsymbol{\omega} \cdot \mathbf{d}_3$ is required to control the ambiguity. Evaluation of the limits of θ will aid in describing attitude motion through a separatrix. These are obtained for the two regimes of interest, i.e., before and after separatrix crossing. After this crossing the energy level is $T < H^2/2I_2$ and corresponding upper and lower limits are obtained by Likins[6] as

$$\sin^2 \theta_u = I_2(2I_3T - H^2)/[H^2(I_3 - I_2)]$$

$$\sin^2 \theta_l = I_1(2I_3T - H^2)/[H^2(I_3 - I_1)]$$

Before separatrix crossing the energy level is $T > H^2/2I_2$. Since ω_1 is always nonzero in this situation, $\mathbf{d}_3$ must oscillate such that the limits of θ are supplements of each other, i.e., $\theta_u = \pi - \theta_l$. The limiting values are then obtained from

$$\left.\begin{matrix}(\pi/2) - \theta_l \\ \theta_u - (\pi/2)\end{matrix}\right\} = \arcsin\left[\frac{2I_1I_3T}{H^2(I_1 - I_3)} - \frac{I_3}{I_1 - I_3}\right]^{1/2}$$

The following terms are defined to nondimensionalize final expressions:

$$I_{13} = I_1/I_3, I_{12} = I_1/I_2, T^* = T/T_{\max}$$

where $T_{\max} = H^2/2I_1$. Thus, the limits on θ become

$$\left.\begin{aligned}\theta_u &= \frac{\pi}{2} + \arcsin\left[\frac{1 - T^*}{1 - I_{13}}\right]^{1/2} \\ \theta_l &= \frac{\pi}{2} - \arcsin\left[\frac{1 - T^*}{1 - I_{13}}\right]^{1/2}\end{aligned}\right\}\text{Before}$$

$$\left.\begin{aligned}\theta_u &= \arcsin\left[\frac{T^* - I_{13}}{I_{12} - I_{13}}\right]^{1/2} \\ \theta_l &= \arcsin\left[\frac{T^* - I_{13}}{1 - I_{13}}\right]^{1/2}\end{aligned}\right\}\text{After}$$

Consider a general case in which $I_1 < I_2 < I_3$. The profile of nutational motion as energy dissipates is illustrated in Fig. 2. Separatrix crossing is shown as a critical point in attitude reorientation after which nutational amplitude decreases to establish spin about the positive or negative major axis, i.e., final nutation angle of 0° or 180°, respectively.

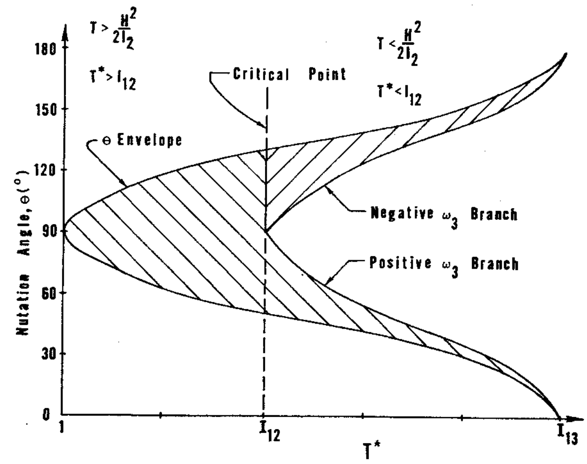

Fig. 2 General nutation profile.

Control Implementation

From the above considerations it is apparent that the critical point in the motion is when energy satisfies $T = H^2/2I_2$ or $T^* = I_{12}$. Nutation angle limits or the sign of ω_3 defines the motion thereafter. Since the sign of ω_3 is easily measurable this will be used as a control input. In addition, magnitudes of $\boldsymbol{\omega}$ components will be used. Active control is engaged as T^* equals I_{12}.

The concept of moving internal masses is applied to lower the inertia ratio I_{12} at the critical point, in order to extend separatrix crossing to the other side. Then these masses are returned to their initial positions, bringing I_{12} back to its original value. Spinning up an internal mass about an axis can increase mechanical energy and alters the distribution of angular momentum. This has essentially the same effect as the moving mass technique. Reaction jets can also be used for this application. Both energy and angular momentum may be controlled. However, application of torques will generally effect the final orientation and magnitude of the angular momentum vector, unless corrective torques are applied later. Selection of control elements will depend on the particular situation at hand.

The moving mass concept is discussed in detail and was modelled as shown in Fig. 3. It is essential to determine sufficient conditions to guarantee a lower I_{12} to prevent separatrix crossing at the critical point. These are developed from a consideration of energy and momentum conditions before and after mass shift. Noting that angular momentum

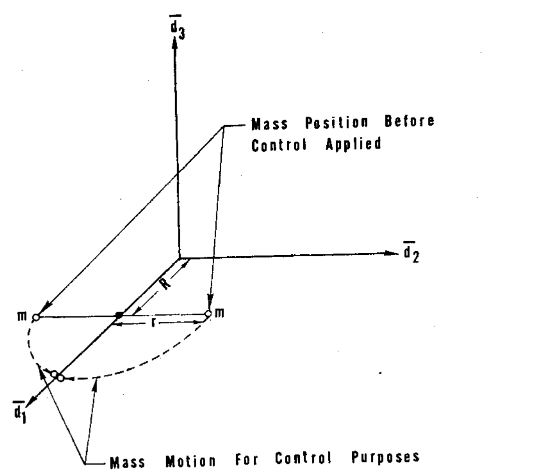

Fig. 3 Example moving mass configuration.

is conserved and the required condition is $T_a^* > I_{12a}$, then the requirement on inertias and angular rates is

$$I_{1b}\omega^2_{1b}\left[\frac{I_{1b}I_{2a} - I_{2b}I_{1a}}{I_{1a}I_{2a}}\right] + I_{3b}\omega^2_{3b}\left[\frac{I_{3b}I_{2a} - I_{2b}I_{3a}}{I_{3a}I_{2a}}\right] > 0$$

By proper selection of the moving mass configuration, the following can be achieved:

$$I_{1b}I_{2a} - I_{2b}I_{1a} > 0, \quad I_{3b}I_{2a} - I_{2b}I_{3a} > 0$$

Replacing I_{1a}, I_{2a}, and I_{3a} with their expanded expressions yields

$$I_{1b}(2R + r) + rI_{2b} > 0, \quad I_{3b}(2R + r) - RI_{2b} > 0$$

The first condition is always satisfied, because all quantities are positive. Since $I_{3b} > I_{2b}$ as specified in the problem statement, the second condition is satisfied for all positive values of R. Thus, the selected mass shift configuration will insure $T_a^* > I_{12a}$.

Simulation and Results

To verify the feasibility of the moving mass system, a simulation was prepared based on the spacecraft described in Ref. 2. This briefly is a vehicle having moments of inertia of 2380, 6940, and 9250 kg $-$ m^2 about the $\mathbf{d}_1$, $\mathbf{d}_2$, and $\mathbf{d}_3$ axes, respectively. A fluid ring damper of radius 1.33m and cross-sectional area 0.48×10^{-2} m^2 is assumed mounted in the 1-2 plane, with its center of gravity at the origin of the body fixed coordinate system. Completely filling the fluid damper then permits an accurate simulation of the dynamics using Euler's equations with an additional equation to allow for energy dissipation.

Assuming elementary pipe flow the friction of the walls is given by

$$\tau_0 = (f/4)\rho v^2/2$$

Using Blasius' empirical friction coefficient of

$$f = 0.316/R_n^{1/4}$$

the total torque on the fluid slug is given by

$$N_3 = \frac{-0.0395\rho R_d^{15/4} P\beta|\dot{\alpha}|^{11/4}}{(D/\nu)^{1/4}\dot{\alpha}}$$

where the $\dot{\alpha}$ term must be maintained in the denominator to insure the proper sign of the torque. Calculation of the kinetic energy of the fluid slug and application of the Lagrangian formulation results in the following equation for the fluid slug acceleration:

$$\ddot{\alpha} = \frac{-0.0395\rho R_d^{15/4} P\beta|\dot{\alpha}|^{11/4}}{(D/\nu)^{1/4}\dot{\alpha}} - \dot{\omega}$$

Combining this with Euler's equations results in the following expressions for motion before the weight shift:

$$\dot{\omega}_1 = \frac{\omega_2\omega_3\{I_2 + A - I_3 - C - 2mr^2\} - C\omega_2\dot{\alpha}}{I_1 + A + 2mr^2}$$

$$\dot{\omega}_2 = \frac{\omega_1\omega_3\{I_3 + C - I_1 - A + 2mR^2\} + C\omega_1\dot{\alpha}}{I_2 + A + 2mR^2}$$

$$\dot{\omega}_3 = \frac{\omega_1\omega_2\{I_1 + 2m(r^2 - R^2) - I_2\} + \dfrac{0.0395\rho R_d^{15/4} P\beta|\dot{\alpha}|^{11/4}}{(D/\nu)^{1/4}\dot{\alpha}}}{I_3 + 2m(r^2 + R^2)}$$

$$\ddot{\alpha} = -\frac{0.0395\rho R_d^{15/4} P\beta|\dot{\alpha}|^{11/4}}{C(D/\nu)^{1/4}\dot{\alpha}} - \dot{\omega}_3$$

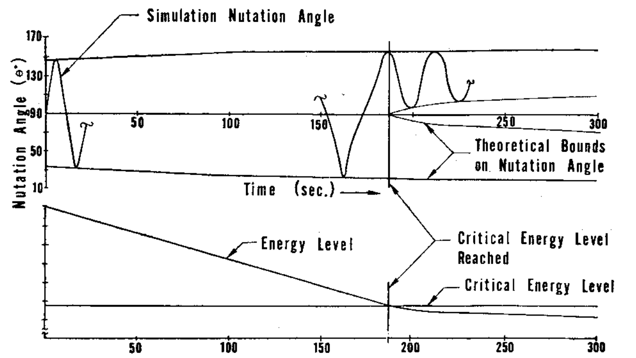

Fig. 4 Uncontrolled nutation near critical point.

Subsequent to the weight shift the equations for $\dot{\omega}_1$, $\dot{\omega}_2$, and $\dot{\omega}_3$ become

$$\dot{\omega}_1 = \frac{\omega_2\omega_3\{I_2 + A - I_3 - C\} - C\omega_2\dot{\alpha}}{I_1 + A}$$

$$\dot{\omega}_2 = \frac{\omega_1\omega_3\{I_3 + C - I_1 - A + 2m(r + R)^2\} + C\omega_1\dot{\alpha}}{I_2 + B + 2m(r + R)^2}$$

$$\dot{\omega}_3 = \frac{\omega_1\omega_2\{I_1 - I_2 - 2m(r + R)^2\} + \dfrac{0.0395\rho R_d^{15/4} P\beta|\dot{\alpha}|^{11/4}}{(D/\nu)^{1/4}\dot{\alpha}}}{I_3 + 2m(r + R)^2}$$

Numerical integration was used to solve these equations. Figure 4 shows the results of the simulation with no control applied. Parameter values used in the simulation are listed below:

$\rho = 1.36 \times 10^4$kg/m^3, $\nu = 1.16 \times 10^{-7}$ m^2/sec, $R = 0.30$m

$D = 0.08$ m, $r = 0.91$ m, $m = 2.26$ *kg*, $R_d = 1.33$ m, $\beta = 2\pi$

Figure 5 shows the same case with the moving mass control system implemented. The quantity T is seen to decrease until the critical value is reached. The negative sign of ω_3 results in activation of the masses. To avoid the requirement of simulating coupling effects during deployment of these masses, motion is assumed rapid when compared to body angular rates. Because of new moments of inertia, the critical energy state is shifted, and although there is a corresponding drop in energy, it is still above the critical level, as predicted by the theory. When ω_3 becomes positive, the masses are returned to their original positions. Energy added by this reset action is quickly dissipated and the angular velocity vector proceeds in the desired direction. It should be noted that a rapidly spinning body with very low-energy dissipation rates could lead to unreliable operation of the control device. If the body

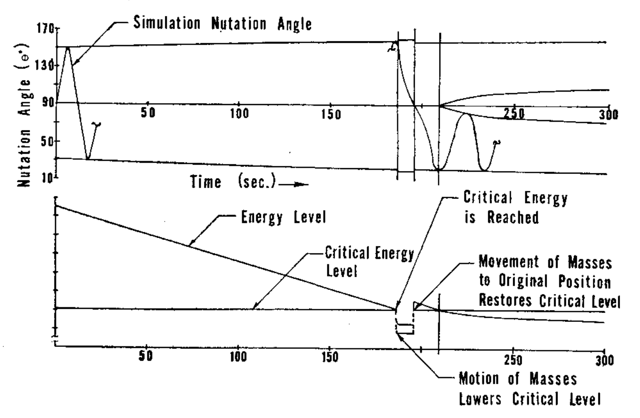

Fig. 5 Controlled nutation near critical point.

remains near the critical energy state for a period of time, angular rate sensors could show several changes of sign for ω_3. However, this situation should be avoidable through appropriate selection of devices and sensors.

It can be seen that the size and location of the control masses are important quantities, as these dictate the size of the steps in the energy state and critical energy level. Sufficient change in I_{12} must be generated to insure that after weight shift T^* does not reach I_{12} before ω_3 changes sign. Also, the dissipative mechanism used in this simulation is especially effective as ω_3 changes sign. Thus, as the weights are returned with ω_3 going from minus to plus, the added energy is quickly dissipated. Different damping systems might require that care be taken not to drive the energy to an excessively high value after the weights have been returned to their original positions. No attempt has been made here to quantitatively describe these limiting parameter values for this situation.

Conclusions

The results of this study indicate that a spinning body can be reoriented from spin about the minor to spin about the major axis using passive energy dissipation, utilizing one of three methods to control the final spin direction. Of these three methods, two (the moving masses and spinning of internal inertia) result in no change in the inertial angular momentum vector. Thus, final state is completely dictated by the initial configuration. The mass expulsion method, while probably the most flexible, generates some change in the angular momentum vector, which would have to be evaluated after completion of the reorientation maneuvers.

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Motion of a Dual-Spin Satellite during Momentum Wheel Spin-Up

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The stability of a dual-spin satellite system during the momentum wheel spin-up maneuver is treated both analytically and numerically. The dual-spin system consists of a slowly rotating main body; a momentum wheel (or rotor) which is accelerated by a torque motor to change its initial angular velocity relative to the main part to some high terminal value; and a nutation damper. A closed form solution for the case of a symmetrical satellite indicates that when the nutation damper is physically constrained from movement (i.e., by use of a mechanical clamp) the magnitude of the vector sum of the transverse angular velocity components remains bounded during the wheel spin-up under the influence of a constant motor torque. The analysis is extended to consider such effect as the motion of the nutation damper during spin-up, and the effect of a nonsymmetrical mass distribution. An approximate analytical solution using perturbation techniques is developed for a slightly asymmetric main spacecraft. For the case of small mass asymmetry the system behaves similarly to the case of a symmetrical satellite; whereas for large asymmetry the frequency change in both the angular velocity components is noted. When the effect of the misalignment of the main spacecraft (spin) principal axis from the geometrical (polar) axis of symmetry is considered, a problem of stability could arise due to the large initial amplification of the system nutation angle.

Nomenclature

A, B, C = moments of inertia about X, Y, Z axes, respectively, for the main body
A', B', C' = A, B, C + rotor contribution
$\bar{A}, \bar{B}, \bar{C}$ = A', B', C' + damper contribution
$C(x,b)$ = Boehmer integral appearing in approximate perturbation solution
$I(1), I(2), I(3)$ = functions appearing in the particular part of the approximate perturbation solution
I_{ij} = moment of inertia tensor of satellite main body $i, j = x, y, z$
K = restoring spring constant of the torsion wire support
k = damping rate constant
l = height of damper plane above X, Z plane
L_i = applied external torque about the quasi-coordinate axis of symmetry
L_{R_y} = rotor torque
M = the mass of the main satellite
$\bar{M}$ = the total system mass $= M + \sum_i^n m_i$

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